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Foundations of Decision Analysis: A Simplified Exposition

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This work was supported by the U.S. Department of Energy, Office of Safeguards and Security.

Photocomposition by E. Katherine Valdez

LA-10702-MS

UC-13
Issued: November 1986

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FOUNDATIONS OF DECISION ANALYSIS: A SIMPLIFIED EXPOSITION

by

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ABSTRACT

Any evaluation procedure requires the specification of evaluation criteria that are critical to the achievement of the objective and that are representative of the system under evaluation. These criteria are expressed numerically as performance measures. The performance measures usually have dissimilar units, and a problem arises in finding a means of relating them to a common unit of measure. Once related to a common unit, they can be aggregated to produce a single scalar of overall system worth. This report presents a simplified exposition of decision analysis, which is a structured approach for evaluating complex alternatives that provides an overall measure of system worth. Evaluations are discussed under situations where the evaluator(s) knows for certain what the outcome will be for any course of action taken and for cases where the outcome is uncertain but can be estimated. Decision making under certainty is covered first, including the concepts of "total value," "value functions," "weights," and "group decisions." Then, decision making under uncertainty is discussed. Included are the parallel topics of "total expected utility," "utility functions," "scaling constants," and "group decisions." Extensions of the procedures described in this report, including fuzzy set theory and optimization methods, are discussed briefly.

I. INTRODUCTION

Complex design, evaluation, and selection problems are often characterized by multiple conflicting objectives, intangible factors, uncertainties, and often several decision makers. The label "decision analysis" refers to the techniques that provide a prescriptive or normative approach for making decisions under these conditions.

Decision analysis is a rapidly growing discipline. Within the last two decades, it has been formally recognized by corporations, governmental bodies, professional societies, and academic institutions. The subject is not difficult, but, unfortunately, much of the formal mathematical work is difficult to understand. We present here a simplified exposition of the foundations of decision analysis.

After a brief discussion of some important concepts, we examine decision making when all possible outcomes are assumed to be known with certainty. Within this topic, we deal with the concepts of "total value," "value functions," "weights," and "group decisions," and provide an example. We next examine decision making under uncertainty. Included in this treatment are the parallel topics of "total expected utility," "utility functions," "scaling constants," and "group decisions" followed by an example. Finally, we discuss briefly the application and extensions of the topics included. For examples and additional explanatory material, see Refs. 1-69.

II. BASIC CONCEPTS

Every evaluation procedure needs one or more evaluation criteria for comparing characteristics or attributes of alternatives. Numerous methods have been proposed for selecting a few criteria that are critical and representative of the system. The criteria represent the major areas of importance and need to be well defined. These criteria are expressed numerically as performance measures. The performance measures usually have dissimilar units, and a problem arises in finding a common unit of measure. For any alternative, the performance measures will be expressed as numbers or expressions representing levels of performance. However, in many cases, the actual worth of a particular performance level is not the number itself but a subjective value judgment made by the evaluator. Thus, for different people, the perceived value may differ.[70]

There are two categories of multiple-criteria decision problems: decisions under (1) certainty and (2) uncertainty. For decisions under certainty, the evaluator knows the outcome for each alternative. For decisions under uncertainty, he knows, or can estimate, a probability distribution for the outcome of each alternative. In decision problems under certainty, the relationships between levels of performance and the subjective worths of these levels are usually called value functions. In decision problems under uncertainty, these relationships are called utility functions. (The term "worth" is used here to designate the numbers associated with either value or utility.) These transformations of performance levels to either values or utilities give all performance measures a common unit of measure. Finally, the value or utility functions can be combined to produce a single scalar of overall system worth.

It is assumed that, for all practical purposes, the set of performance measures adequately represents the set of criteria and the levels of performance can be specified. In the following sections, we examine first the case of multiple-criteria decision making under certainty followed by decision making under uncertainty.

III. DECISION MAKING UNDER CERTAINTY

Assume there are k alternatives, or courses of action, to be evaluated. Let these be designated by $A = \{a_i | i = 1, ..., k\}$. In addition, there are m criteria that characterize the system, which are designated by $C = \{c_i | i = 1, ..., m\}$. Each of these c_i's can be further subdivided to produce lower-level subcriteria. Finally, for each lowest-level subcriterion, a performance measure is specified that measures the degree to which its associated subcriterion is satisfied. There is no loss of generality if we consider only the lowest-level criterion involved in the decision problem, with the understanding that many problems involve more than one level and the system value is aggregated from the lowest to the highest level. Working at the lowest level and assuming C is the set of subcriteria at that level, we find that the m performance measures for these subcriteria are represented by $X = \{X_i | i = 1, ..., m\}$. A specific set of numerical values of the performance measures, referred to as levels of performance, can be displayed in the vector form $\underline{x} = (x_1, x_2, ..., x_m)$.

A. Total Value

The total system value can be defined in many ways. An additive model can provide a good approximation without significantly decreasing the accuracy of the total value.[71,72] Under general conditions of independence (to be discussed later) the additive model is completely justified, and the total system value is

$$v(x_1, x_2, ..., x_m) = \sum_{j=1}^{m} w_j v_j(x_j) \quad , \tag{1}$$

where $v_j(x_j)$ is the one-dimensional value of a particular level of performance, x_j, and w_j is a positive scaling constant called a weight. The x_j's, $v_j(x_j)$'s, and w_j's are numbers. The choice of the range of the w_j's and $v_j(x_j)$'s is arbitrary, but it is convenient for them to be

$$0 < w_j < 1 \ , \tag{2}$$

$$0 \leq v_j(x_j) \leq 1, \quad \text{for} \quad j = 1, \ldots, m \ , \tag{3}$$

and

$$\sum_{j=1}^{m} w_j = 1 \ . \tag{4}$$

The system with the largest total system value $v(\underline{x})$ is selected.

Three topics requiring explanation are (1) construction and properties of value functions, (2) independence, and (3) determination of criteria weights. Each of these will be discussed, the techniques applied to group decisions, and an example given. However, first let us introduce the concept of a weak ordering, which will be useful in the following discussions. Assume that C is composed of three criteria: c_1, c_2, and c_3. For Eq. (1) to be used, that is, for Eqs. (2) and (3) to be determined, a weak ordering of the criteria must be possible. A weak ordering of criteria means that the criteria are comparable and that the preferences for the criteria are transitive. Also, for Eq. (3) to be valid, we must be able to state in a formal preference structure the relationship of one performance level to other levels of the same performance measure. For our three criteria, transitivity means, for example, that paired comparisons may yield $c_1 \succ c_2$, $c_2 \succ c_3$, and $c_1 \succ c_3$; where $\succ$ means "is preferred to" and gives the ordering c_1, c_2, and c_3. If the ordering is transitive, the third comparison need not be made because it is implied by the first two comparisons. In some cases, an evaluator will be indifferent in preference between criteria. For example, the evaluator may slightly prefer c_1 to c_2 or be indifferent in preference between c_1 and c_2, while at the same time preferring c_1 to c_3. Here, the same ordering would hold as before, and the first comparison would be $c_1 \succeq c_2$, where $\succeq$ would be read "is preferred or indifferent to." That is, criteria can be ranked in order of preference (importance), and any adjacent criteria may be of equal importance. The weak ordering requires only the concept of the ordering by $\succeq$. A preference structure must be established both for determining the weights and for constructing the value functions.

B. Value Functions

A value function is a preference representation function under certainty.[1,73] Thus, a value function is a scalar-valued representation of both the preferences of x_j over x_j^0 (assuming level x_j is preferred to level x_j^0) and the strength of that preference. Mathematically stated, if

$$x_j \succ x_j^0 \ , \tag{5}$$

then

$$v_j(x_j) \geq v_j(x_j^0) \ . \tag{6}$$

Also, if the evaluator is indifferent between x_j and another level, x_j', the values $v_j(x_j)$ and $v_j(x_j')$ are equal. This is stated as

$$x_j \sim x_j' \ ,$$

where $\sim$ is read "is indifferent to;" then

$$v_j(x_j) = v_j(x_j') \ .$$

A value function reflects the degree to which each performance level contributes to the achievement of the objective, or to the satisfaction of the criterion. Therefore, value functions can either increase or decrease as performance levels increase or decrease. Value functions can be represented on an interval scale because they are unique up to a monotonic transformation. If $v_j'(X_j)$ is a value function for X_j, then so is $v_j(X_j)$, if for any positive constant, a_j,

$$v_j(X_j) = a_j v_j'(X_j) + b_j \ .$$

On an interval scale, adjacent values are separated by a constant arbitrary unit of measurement, and the ratio of any two intervals is independent of both the unit of measurement and an arbitrary zero point. Because there is no natural origin, it may be chosen in any convenient manner. If the origin is set to zero, the least-preferred or worst level of X_j is set to zero and the best level to one; then

$$0 \leq v_j(x_j) \leq 1 \ .$$

The next step in constructing a value function requires that, within the end points or range of X_j, enough coordinates be estimated so that a graph can be constructed. By asking qualitative questions, we can determine the general shape of the value function (linear, concave, reverse-S shaped, and so forth). Fishburn[74] presents some methods for estimating value functions and others for estimating weights. In one of these methods, the evaluator estimates the value of the performance level midway between the best and worst levels. This point now separates two intervals; midpoints are estimated for these in a similar manner. This subdividing continues until enough points have been established to plot a graph. Consider $v_1(X_1)$ as an example. Let $x_1^0 = 100$ be the worst level and $x_1^1 = 0$ be the best level. Suppose that questioning has determined that $v_1(X_1)$ is a reverse-S-shaped curve. The midpoint is expressed as

$$v_1(x_1^{0.5}) = 0.5[v_1(100) + v_1(0)] = 0.5 \ ,$$

and $x_1^{0.5}$ must be specified by the evaluator. It is important to realize that

$$v_1(0) - v_1(x_1^{0.5}) = v_1(x_1^{0.5}) - v_1(100) \ ,$$

or that moving from 0 to $x_1^{0.5}$ has the same differential value as moving from $x_1^{0.5}$ to 100. For example, let $x_1^{0.5} = 41$. Next, establish the midpoints from 0-41 and 41-100, and so on.

If there is a natural zero, a ratio scale may be formed; that is, b_j equals zero. This can be justified if the value function measures the fractional attainment of an objective. If a performance measure has no natural or logical upper or lower bound, then a practical operational bound must be specified that represents the limit of what one might reasonably expect to attain. For more detail on construction of value functions, see Refs. 1, 65, 75, and 76.

The most important assumption related to Eq. (1) is that the performance measures are value (or preferentially) independent. This justifies using addition to combine the individual value functions. Loosely speaking, value or preferential independence means that an evaluator's preferences for a specific level of one performance measure do not influence his preferences for levels of any other measure. Although in some cases performance measures can be dependent without affecting results, in other cases, only by carefully combining some and/or eliminating others, can the total value be maintained with minimum distortion. Care in defining criteria and performance measures can eliminate many problems (see, for example, Ref. 77). Precautions taken when the value functions are being constructed also can significantly minimize the effects of interactions. More theoretical expositions on independence are given by Fishburn[74,78,79] and by Keeney and Raiffa.[1]

C. Weights

Finally, we must determine the scaling constants, or weights. We can use several methods for transforming experts' judgments into relative weights. In the constant-sum method discussed here,[80] 100 points are distributed between every pair of criteria at the same level. If P_{jk} is the number of points awarded to c_j when c_j is compared with c_k, the ratio of points for c_j and c_k is given by

$$r_{jk} = \frac{P_{jk}}{P_{kj}} , \quad j, k = 1, ..., m , \quad j \neq k ,$$

where $0 < P_{jk} < 100$, and $P_{kj} = 100 - P_{jk}$.

If there are m criteria, $m(m - 1)/2$ paired evaluations are conducted, even though only $m - 1$ are needed to construct the weights. However, we can use all estimates to check for inconsistencies and to calculate composite scale values. The scale values are given by

$$s_j = \text{antilog}\left(\frac{1}{m} \sum_{k=1}^{m} z_{jk} \right) , \quad j = 1, ..., m ,$$

where z_{jk} is the log of r_{jk}. The s_j's have a geometric mean of 1 and are converted to the normalized form by

$$w_j = \frac{s_j}{\Sigma s_k} , \quad j, k = 1, ..., m .$$

Because the constant-sum method uses comparisons of all pairs, it can be used to check on transitivity and to obtain indirect estimates of the weights. For additional discussions on weighting, see Refs. 1, 65, 72, and 81-89; and for additional weighting methods, see Refs. 54 and 90-95. Although some other authors, including Keeney and Raiffa,[1] disagree with this method of determining weights, most weighting methods described in the literature are similar to it.

D. Group Decisions

Equation (1) can be used with one or more evaluators. In many cases, a group of evaluators will be involved in constructing value functions and determining weights. However, although in this case it is common to produce group weights and group value functions and use them in Eq. (1), before this can be done, some form of group consensus must be reached.

A popular method for determining group consensus uses the Kendall Coefficient of Concordance,[96] which holds that a significant value for the coefficient suggests probable agreement. According to Kendall, if there is agreement, the best estimate of the correct group ranking is given by the order of the sums of the ranks.[96,97] However, other authors disagree with this procedure when certain conditions hold.[98] If there is consensus, the individual weights and value functions are either averaged or developed by group discussions or Delphi-like techniques.[99,100] For more information on group consensus, see Refs. 101-115.

Not everyone agrees that a group evaluation can be conducted by using one function, such as Eq. (1), for all evaluators. In a pioneering work in 1951, Arrow[116] proposed a set of five reasonable assumptions that should hold for any scheme that aggregates individual rankings into a group ranking. He proved that no aggregation scheme was compatible with all five assumptions. These assumptions, along with the proof that there is no rule for combining the individual rankings that is consistent with the assumptions, are known as Arrow's Impossibility Theorem. Because Arrow considered rankings, no strengths of individual preference or of interpersonal preference comparisons were included. Keeney[117] proved that group preferences can be aggregated if value functions and utilities rather than rankings are used. The group function thus derived by using assumptions analogous to Arrow's is similar to Eq. (1), except that m represents the number of evaluators, v_j is a value function (under certainty) or a utility function (under uncertainty) for the j^{th} individual, and w_j is a positive scaling constant determined by examining interpersonal comparisons. A comprehensive discussion of Keeney's work on the aggregation of individual preferences is presented in Ref. 1.

E. Example

The concepts discussed above are illustrated in the following evaluation of a volume-reduction process for radioactive waste. The evaluation criteria are effectiveness, flexibility, availability, operability, and resource use. Effectiveness is the ability of the process to meet the volume-reduction objective, and the performance measure for effectiveness is the volume-reduction factor. Flexibility is the ability of the process to accept a wide range of waste compositions at different processing rates. Flexibility is determined by how well the process handles waste in the four (subcriteria) categories: normal components, solids throughput, noncombustibles, and liquid throughput. Normal components include PVC, paper and rags, polyethylene, rubber, and dense cellulosics. A composite formula relating the percentage of each component in the feed and the feed rate is used as a performance measure. The performance measure for solids throughput, for example, is the rated capacity (kg/h) of the unit. Availability is the frequency with which the process operates satisfactorily and could be directly measured by calculating the percentage of uptime. However, because engineers are more familiar with downtime, that measure is used instead. Operability is the level of operational complexity in terms of equipment, controls, and manpower. The operability performance measure is the minimum number of parameters that must be controlled for the process to operate and meet all requirements.

Resource use involves the following subcriteria: energy, scarce strategic materials, water, and land. Performance measures for these subcriteria are expressed according to their degree of use—extensive, high, moderate, etc. Figure 1 shows the relationships between the top two criteria levels. All Kendall Coefficients of Concordance were significant at the 0.05 level or better (see Ref. 96 for the meaning of significance).

Ten process engineers used the constant-sum method to determine weights, which were then averaged and presented to the group for modification/agreement. The list below shows the average weights for the final main (upper-level) criteria.

Criterion	Weight
Effectiveness	0.35
Flexibility	0.22
Availability	0.20
Operability	0.13
Resource use	0.10
	1.00

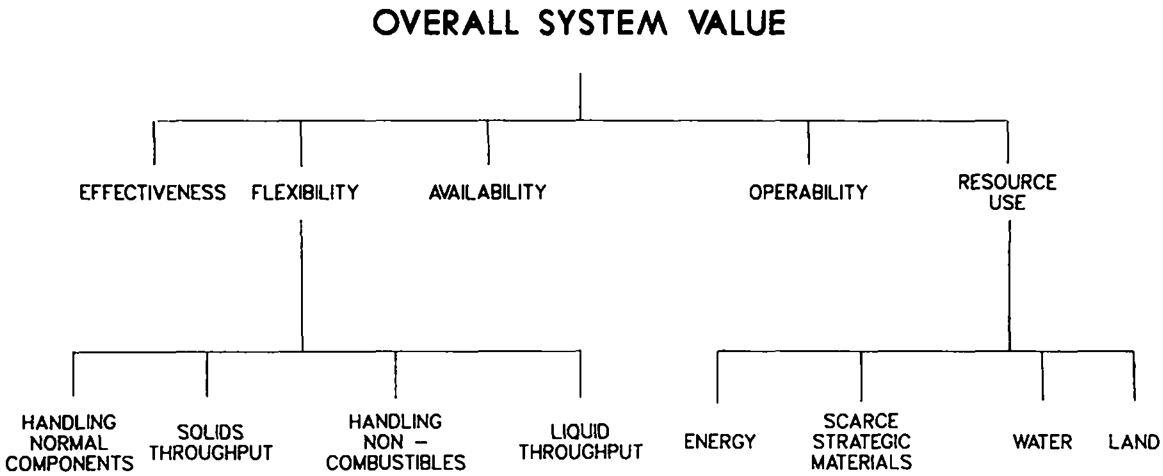

Fig. 1. Criteria structure.

Each evaluator was asked a series of questions to determine the shapes of the value functions. Then, each one sketched a curve representing his perception of the value function for each performance measure. After checking for consistency, the engineers averaged the corresponding value functions to produce group value functions and presented each to the group for modification/agreement. The final group value function for the volume reduction factor is shown in Fig. 2.

The evaluation procedure involves determining the specific levels of performance for each alternative being evaluated, finding the corresponding values from the appropriate value functions, and then aggregating all values by multiplying by the corresponding weights (see list above) and by adding all terms.

For example, if the volume reduction of a particular process was 25:1 (performance level for volume-reduction factor), from Fig. 2 we see that the associated value is 0.74. The list above shows the weight for effectiveness as 0.35. We derive the total contribution of effectiveness by multiplying (0.35)(0.74) = 0.26. If the total value of this process were 0.76, effectiveness contributed approximately 34% of the total. A thorough discussion of this example can be found in Ref. 65.

IV. DECISION MAKING UNDER UNCERTAINTY

In decisions under certainty, the alternative a_i with the largest total value is selected. In decisions under uncertainty, the alternative a_j with the highest expected utility is selected. The results shown in the previous section need only a few changes to address decision making under uncertainty. As before, we have A, C, and X representing the sets of alternatives, criteria, and performance measures, respectively. Also, the vector $\underline{x}$ can be specified, but with uncertainty about the individual x_i's.

A. Total Expected Utility

Let the total system's expected utility be

$$u(x_1,x_2,\ldots,x_m) = \sum_{j=1}^{m} w_j u_j(x_j) \ , \tag{7}$$

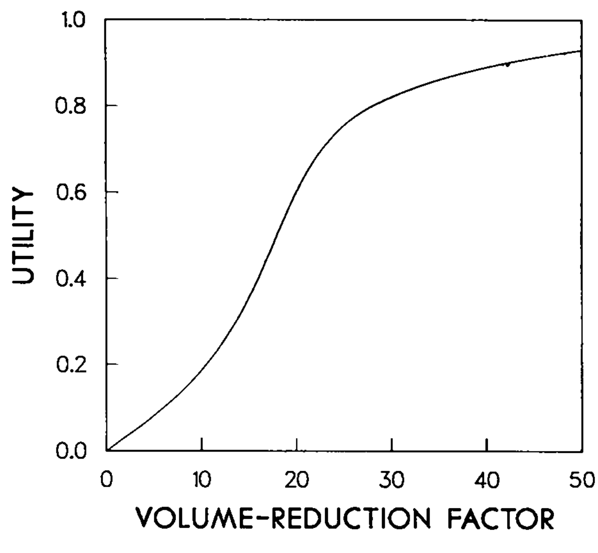

Fig. 2. Group value function for the volume-reduction factor.

where $u_j(x_j)$ is the one-dimensional utility of a particular level of performance, x_j, and w_j is a positive scaling constant. Equations (2) and (4) still hold, and

$$0 \leq u_j(x_j) \leq 1 , \quad \text{for} \quad j = 1, \ldots, m . \tag{8}$$

Construction and properties of utility functions, independence, and determination of the scaling constants will be discussed in turn, followed by a discussion of group decisions and an example.

B. Utility Functions

A utility function is a value function, but the opposite is not necessarily true. If we have a value function, a utility function can be constructed by incorporating the element of uncertainty. Alternatively, if a value function does not exist, the utility function can be derived directly. Most techniques involve the use of lotteries in questioning the evaluator on preferences for specific levels of performance.

Assume that the criteria can be weakly ordered and a preference structure can be stated for each X_j. Also, if x_j is preferred to x_j^0, then

$$u_j(x_j) > u_j(x_j^0) . \tag{9}$$

Also, when either a set of probabilities (if x_j is discrete) or a probability density function (if x_j is continuous) can be associated with $\underline{x}$, a utility function can be constructed satisfying Eqs. (8) and (9). Consider another level, x_j^1. Let $x_j^1 > x_j$. Then, $u_j(x_j^1) > u_j(x_j) > u_j(x_j^0)$. However, at some point, an evaluator would be indifferent between obtaining x_j for certain and a lottery L (gamble) with a π chance at the more preferred x_j^1 and a $1 - \pi$ chance at the less preferred x_j^0. This lottery is represented notationally by

$$L[\pi, x_j^1; (1 - \pi), x_j^0] .$$

The indifference relationship is represented by

$$x_j \sim L[\pi, x_j^1; (1 - \pi), x_j^0] .$$

Furthermore, the evaluator would be indifferent between obtaining x_j^1 for certain and the lottery only if $\pi = 1$. Also, he would always prefer a lottery involving x_j^0 to obtaining x_j^0 with certainty unless $\pi = 0$. The interpretation of π is the probability of obtaining x_j^1, where high values associated with π will tend to have the evaluator select the lottery, rather than the certainty of obtaining x_j. If x_j^1 is the best or highest level achievable, then $u_j(x_j^1) = 1$, and if x_j^0 is the lowest level, $u_j(x_j^0) = 0$. Now there must be some intermediate utility $u_j(x_j)$ for x_j. From the above discussion and because $u_j(x_j)$ must equal the expected utility of the lottery,

$$u_j(x_j) = \pi u_j(x_j^1) + (1 - \pi) u_j(x_j^0) .$$

Solving for π, we see that

$$\pi = u_j(x_j) .$$

Remember, π is not the probability that x_j^1 will occur; it is a lottery ticket suggesting the evaluator's preferences for x_j^0, x_j, and x_j^1. (However, for every alternative in A, each performance measure is covered by a probability distribution.) The utility function can be constructed by repeating the lottery several times with judiciously chosen levels of performance.

Utility functions are constructed from value functions by a method similar to the one described above. For example, if $v_j(x_j^1) = 1$, $v_j(x_j^0) = 0$, and π is the chance of obtaining $v_j(x_j)$ in a lottery, then $u_j[v_j(x_j)] = \pi$. This lottery would be repeated until enough points were available to plot the utility function.

As with decision under certainty, some type of independence must hold if we are to use an additive model. Earlier we described preferential or value independence. For decision under uncertainty, we are concerned additionally with utility independence and additive independence. Assumptions of utility and additive independence not only simplify the combination of separate utility functions and the construction of individual utility functions but also can be verified by questioning evaluators when the individual functions are constructed. Let us consider first utility independence and then additive independence.

Preferential independence involves tradeoffs among different performance measures, whereas utility independence involves preferences for lotteries with different performance levels for one performance measure and fixed levels for other performance measures. Dependence, then, is present when preferences for lotteries over one or more performance measures depend on levels of other performance measures.

Let X_i represent performance measure i and X_{i-} represent the remaining performance measures, or complement of X_i. Then X_i is utility independent of X_{i-}, provided that preferences for all lotteries on X_i that are defined over the performance levels x_i, with the levels of X_{i-} fixed, are unaffected by the amounts of x_{i-}. This conclusion implies that the conditional utility function over X_i, given X_{i-} fixed at any value x_{i-}, is a positive linear transformation of the conditional utility function of X_i for some other level of X_{i-}. Thus, a conditional utility function is a utility function evaluated over X_i with X_{i-} held fixed.

For simplicity, consider the two performance measures X_1 and X_2. If $u(x_1,x_2)$ is a utility function and X_1 is utility independent of X_2, preferences for lotteries on X_1 and X_2 do not depend on the particular level of X_2, say x_2. Therefore,

$$u(x_1,x_2) = a + bu(x_1,x_2^0) \tag{10}$$

for all x_1 and x_2, where a and b are constants and $b > 0$. If, in addition, X_2 is utility independent of X_1, they are called mutually utility independent. If X_1 and X_2 are mutually utility independent,

$$u(x_1,x_2) = u(x_1^0,x_2) + u(x_1,x_2^0) + Ku(x_1^0,x_2)u(x_1,x_2^0) \tag{11}$$

for some constant K.

Another representation of Eq. (11), given as Eq. (13), is obtained by setting

$$u(x_1^0,x_2^0) = 0 \quad \text{and} \tag{12}$$

$$u(x_1^1,x_2^1) = 1$$

and using Eq. (10) and some algebraic manipulation, and assuming mutual utility independence.

$$u(x_1,x_2) = k_1u_1(x_1) + k_2u_2(x_2) + k_{12}u_1(x_1)u_2(x_2) \ . \tag{13}$$

Because

$$u_1(x_1^0) = u_2(x_2^0) = 0 \ , \qquad \text{and} \qquad u_1(x_1^1) = u_2(x_2^1) = 1 \ ,$$

and because utility functions are unique up to a positive linear transformation,

$$u(x_1,x_2^0) = k_1u_1(x_1) \ , \qquad \text{and} \tag{14}$$

$$u(x_1^0,x_2) = k_2u_2(x_2) \ . \tag{15}$$

Also,

$$k_1 = u(x_1^1,x_2^0) \ , \qquad k_2 = u(x_1^0,x_2^1) \ , \qquad k_{12} = 1 - k_1 - k_2 \ , \qquad \text{and}$$

$$K = k_{12}/(k_1k_2) \ ,$$

where k_1 and k_2 are positive scaling constants. The proof or derivation of Eq. (13) is straightforward and can be found in Ref. 1.

Mutual utility independence is a necessary but not a sufficient condition for additive utility independence. Additive utility independence, however, does imply mutual utility independence. If, for X_1 and X_2, preferences over the two lotteries

$$L_1[0.5,(x_1,x_2); 0.5,(x_1^0,x_2^0)] \quad \text{and}$$

$$L_2[0.5,(x_1,x_2^0); 0.5,(x_1^0,x_2)]$$

depend only on their marginal probability distributions and not on their joint distribution, X_1 and X_2 are additive independent. If X_1 and X_2 are additive independent, the evaluator is indifferent between L_1 and L_2 because they have the same marginal probability distribution. Because of the indifference between L_1 and L_2, their associated expected utilities are equal or

$$0.5u(x_1,x_2) + 0.5u(x_1^0,x_2^0) = 0.5u(x_1,x_2^0) + 0.5u(x_1^0,x_2) \ .$$

From Eqs. (12), (14), and (15), we find

$$u(x_1,x_2) = k_1u_1(x_1) + k_2u_2(x_2) \ , \tag{16}$$

with k_1 and k_2 as before.

Equation (13) is the same as Eq. (16) except for the last term in Eq. (13). Therefore, if X_1 and X_2 are mutually utility independent, the utility function $u(x_1,x_2)$ can be expressed either as an additive function when $K = 0$ or as a multiplicative function when $K \neq 0$. (The above discussion is based on Refs. 1 and 79.)

The results shown to hold for the two performance measures X_1 and X_2 can be demonstrated to hold for X_i and X_{i-}. These results will be stated without much discussion.

The pair of performance measures X_i and X_j is said to be preferentially independent of X_{ij-}, where X_{ij-} is the complement of X_{ij}, if the preference order for the pair x_i, x_j with x_{ij-} fixed does not depend on the fixed x_{ij-}. If X_i is mutually utility independent of X_{i-}, and X_i and X_j are preferentially independent, $u(\underline{x})$ must be either the additive form or the multiplicative form. The additive form is

$$u(x_1,x_2,\ldots,x_m) = \sum_{i=1}^{m} k_iu_i(x_i) \ , \tag{17}$$

with

$$\sum_{i=1}^{m} k_i = 1 \ ,$$

where $k_i = w_i$ from Eq. (7). The multiplicative form is

$$1 + Ku(x_1,x_2,\ldots,x_m) = \prod_{i=1}^{m} [1 + Kk_iu_i(x_i)] \ , \tag{18}$$

with

$$\Sigma k_i \neq 1 \ , \quad \text{and} \quad -1 < K < 0 \ .$$

If $K = 0$ in Eq. (18) or if $\Sigma k_i = 1$, Eq. (18) reduces to Eq. (17). Setting all X_i's to their best levels in Eq. (18) produces

$$1 + K = \prod_{i=1}^{m} (1 + Kk_i) \ , \tag{19}$$

which can be solved for K. Equation (17) corresponds to Eq. (7), and Eq. (18) is a new form. (The above discussion was based on Refs. 1 and 118-120.)

C. Scaling Constants

The scaling constants, k_i, are determined by asking the evaluator to compare a performance measure at its best level and all the others at their worst levels with a lottery with all performance measures at their best levels with probability of occurrence (under the lottery) π or with all performance measures at their worst levels with probability of occurrence $1 - \pi$. The objective is to find the value of π where the evaluator is indifferent between the lottery and the certainty case. This is stated as finding π_i so that the evaluator is indifferent between

$$(x_1^0,\ldots,x_{i-1}^0,x_i^1,x_{i+1}^0,\ldots,x_m^0)$$

and the lottery $L[\pi_i,x_i^1; (1 - \pi_i),x_i^0]$, where x_i^j $(j = 0,1)$ is the vector of performance levels. The expected utility of the lottery is equal to the utility of the certainty case or

$$u(x_i^1,x_{\bar{i}}^0) = \pi_i u_i(x_i^1) + (1 - \pi_i)u_i(x_i^0) \ . \tag{20}$$

Substituting for $u(x_i^1)$ and $u(x_i^0)$, we find

$$u(x_i^1,x_{\bar{i}}^0) = \pi_i \ ,$$

but

$$u(x_i^1,x_{\bar{i}}^0) = k_i \tag{21}$$

from Eq. (17) or (18). Assume the k_i's are ranked and the largest, say k_1, is determined as described above. The remaining k_j's can be related to the largest scaling constant, k_1, by questioning the evaluator about preferences for a level of X_i, say x_i', and a level of X_j, say x_j', so that he is indifferent between

$$(x_i',x_j^0,x_{\bar{ij}}^0) \text{ and } (x_i^0,x_j',x_{\bar{ij}}^0) \ . \tag{22}$$

Therefore,

$$k_i u_i(x_i') = k_j u_j(x_j') \ . \tag{23}$$

If $x_j' = x_j^1 (i = 1 \text{ and } j = 2)$, we find $k_2 = k_1 u_1(x_1')$, where $u_1(x_1')$ is easily found for x_1' if $u_1(X_1)$ is available. Next, another level of X_1, say x_1'', is selected and a comparison of preferences is conducted with x_3^1 $(j = 3)$ so that the evaluator is indifferent between $(x_1'',x_3^0,x_{\bar{13}}^0)$ and $(x_1^0,x_3^1,x_{\bar{13}}^0)$. Therefore, $k_3 = k_1 u_1(x_1'')$. We proceed in this manner until all k_i's are determined.

If $\Sigma k_i = 1$, Eq. (17) is used; otherwise, Eq. (18) is used. When Eq. (18) is appropriate, Eq. (19) can be solved iteratively until K converges. If $1 < \Sigma k_i$, then $-1 < K < 0$; and if $\Sigma k_i < 1$, then $0 < K$.[1] The additive form of the utility function given by Eq. (17) holds when additive independence conditions exist for X. Additive independence involves indifference between lotteries, whereas utility independence involves indifference between a lottery and its certainty equivalent. The multiplicative form given by Eq. (18) holds when mutual utility independence conditions exist for X. Remember that additive utility independence implies mutual utility independence. This leads to some interesting interpretations of K. According to Keeney[119] and Keeney and Raiffa,[1] K indicates how the amount of one performance measure affects the value of another performance measure. For the two performance measures X_1 and X_2, they can be either substitutes when $K < 0$ or complements when $0 < K$. When $K = 0$ there is no interaction of preferences between X_1 and X_2. Selection of either Eq. (17) or Eq. (18) for use with an actual problem depends on K. Procedures for choosing the correct value of K are presented in Ref. 1.

It is important to check the consistency of the individual conditional utility functions and the scaling constants.[1,121] Checking consistency is not difficult; many procedures to facilitate this are apparent from the above discussion. For example, the procedures for selecting the correct K along with Eq. (19) are one consistency check on K. Another example of consistency checking is that the k_i should be ordered identically to the preference ordering of X. However, k_i does not indicate the degree of relative importance of X_i to any X_j. For a thorough discussion on checking consistency, see Ref. 1.

D. Group Decisions

Finally, we consider the aggregation of n individuals' utilities. Two possibilities can occur that apply to group aggregation of value functions as well as of utility functions. In one case, one final decision-maker (evaluator) combines the utility functions of either a supporting staff or a group of experts. In the other case, the final utility function will be determined for a group of individuals. The group decision problem is more interesting because of differences in preferences and related probabilities. However, in this survey, we assume that individual and group utility functions can be constructed.

Equation (17) with $m = n$ can be considered to be the group utility function with the k_i's acting as positive scaling constants and u_i as the utility function for the i^{th} individual. The multiplicative form given by Eq. (18) has a corresponding group utility function with the same substitutions as mentioned above for Eq. (17). For single evaluators, the k_i's would result from the evaluators' interpersonal comparisons of their individual preferences. If a group decision is involved, the assessment of the k_i's is difficult. One method of determining the scaling constants in the group situation would be to set up some arbitration scheme that would lead to a set of constants agreeable to each group member. However, the conditional utility functions (or value functions), u_i, can be assessed by each individual in the group. Independence assumptions similar to those for one evaluator are required for the group decision problem.

For more on group decisions, see Refs. 1, 43, 46, 56, 73, 117, and 122-127. For two other points of view on individual and group evaluations, see Refs. 128 and 129, respectively.

E. Example

Consider the example given in Sec. III as a hypothetical utility problem. Assume that the ranking of the main criteria is as indicated in the list in Sec. III.E. Also consider a realistic upper bound for the volume-reduction factor to be 50 giving $u_1(50) = 1$. Likewise, $u_1(0) = 0$. The utility function, $u_1(x_1)$, was assessed directly by using certainty equivalents. First we determined the central point, that is, the certainty equivalent $x_1^{0.5}$ for the lottery producing either $x_1^0 = 0$ or $x_1^1 = 50$, each with a probability of 0.5. The remaining points were found by determining the certainty equivalents for the midpoints of the two intervals produced by the first lottery. This process was repeated until enough points were available to plot the utility function. The hypothetical utility function is shown in Fig. 3.

To evaluate the scaling constants, the hypothetical evaluator was asked at what point (probability) he would be indifferent between X_1 being at its best level (50) and all other performance measures being at their worst levels and a lottery with all performance measures at their best levels or all at their worst levels. This situation is given symbolically by Eq. (20). Assume this probability is 0.4 for the volume-reduction factor. Then from Eq. (21), $k_1 = 0.4$. Next, the evaluator was asked at what levels of the performance measures X_1 and X_2 would he be indifferent between x_1^1 and x_2^0 together and x_1^0 and x_2^1 together with all other performance measures at their worst levels. This situation is given by Eq. (22).

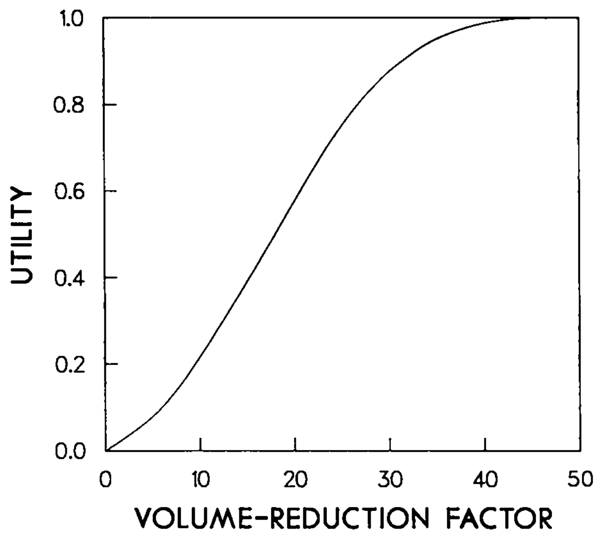

Fig. 3. Hypothetical utility function for the volume-reduction factor.

Also, we assigned $x_2' = x_2^1$ so that Eq. (23) could be used. Here X_2 represents the percentage of downtime, and $x_2^1 = 5\%$ with $u_2(5\%) = 1$. Also, $x_2^0 = 80\%$ with $u_2(80\%) = 0$. The evaluator was searching for x_1' with 80% of downtime and for 0 with 5% of downtime where he would be indifferent between these consequences. Assume he picked a volume-reduction factor of 20($x_1' = 20$). From Fig. 3, $u_1(20) = 0.58$. Therefore, from Eq. (23) and the conditions specified above, $k_2 = (0.4)(0.58) = 0.23$. For the complete example, 15 scaling constants are required for the additive utility function given by Eq. (17). We assume, from questioning evaluators, that additive utility independence holds. The complete specification of the total additive utility function is produced by repeating the steps illustrated above until all the individual utility functions and scaling constants are determined. Then the total utility is combined using Eq. (17) by multiplying the appropriate individual utility functions and scaling constants and summing all the resultng terms.

Assume the total utility for a particular process that has a volume-reduction factor of 25 is 0.84. From Fig. 3, $u_1(25) = 0.75$. The total contribution of effectiveness is $(0.4)(0.75) = 0.3$, or approximately 36% of the total utility.

V. APPLICATION AND EXTENSIONS

Operational considerations dictate that a decision-making procedure be relatively easy to understand and explain, easy to implement, economical, reasonably stable against small errors in estimates, and easy to modify slightly, if necessary. The decision analysis techniques presented here satisfy all of these considerations, are well developed theoretically, and are used in a wide range of operational settings.

A primary benefit of the procedure is that it helps structure a complex evaluation in a systematic way. The arrangement of the criteria and performance measures in a hierarchical structure allows a visual representation of the complex evaluation process. The evaluator can observe at a glance the structural relationships among all components. This visualization can be very important at the early stages of an evaluation and can aid in restructuring the problem, if necessary. Visualization is also important when more than one evaluator is involved. Furthermore, additional alternatives can be synthesized during evaluation. The graphic structure will allow the evaluator to feel comfortable with the evaluation. If he does not feel comfortable, this also is valid information, indicating that something is wrong. The evaluation procedure, if not completely satisfactory, at least forces the evaluator to clarify and quantify factors important to the overall decision. One of the primary benefits of the procedure is that a systematic approach can be taken to quantify the problem, which in turn allows the evaluator to examine his or her choices and to communicate the rationale used.

Pursuing all the steps described above is often unnecessary; at some point, it may be obvious that further analysis is unnecessary. After the objectives, criteria, performance measures, and performance levels are identified, one or two systems may be clearly superior to (dominate) all others. Many inferior alternatives can also be eliminated in a preliminary screening of design alternatives by using a simple deterministic model. The remaining analysis could focus on areas where more refined information is required.

Extensions to the procedures described in this section for multiple-criteria decision problems include fuzzy set theory[130-135] to account for the imprecision or ill-defined nature of some problem components, and optimization methods[136-139] for the generation of efficient solutions when the number of criteria or number of alternatives is large. The performance measures, preference relations, weights, and value (utility) functions can be fuzzy. The rank ordering of the alternatives can also be fuzzy.[130,131] Some of this fuzziness may be due to the imprecision that involves almost all real-world problems, causing evaluators to balk at providing precise estimates of weights, probabilities, or utilities,[132] although they may be willing to state them in terms of linguistic values, such as "moderately important," "very important," and so forth.[134] These linguistic values could be mapped as fuzzy numbers on the closed interval [0,1]. In group-decision problems, consensus can be examined by fuzzy group-preference relations.[135] An added advantage of using fuzzy set theory is that sensitivity analysis is automatically included in the calculations.[132,133] Fuzzy calculus can also be applied to multiple-criteria optimization methods.[140] More information on multiple-criteria optimization, fuzzy applications, and other extensions can be found in international conference proceedings given in Refs. 141-154.

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